

$$2) a) \lim_{n \rightarrow \infty} \frac{e^{\sqrt{3}/n} - 1}{\sin \frac{2}{n}} = \left[ \frac{0}{0} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{e^{\sqrt{3}/n} - 1}{\sin \frac{2}{n}} \cdot \frac{\frac{2}{n}}{\frac{2}{n}} \cdot \frac{\frac{\sqrt{3}}{n}}{\frac{\sqrt{3}}{n}} = \lim_{n \rightarrow \infty} \frac{e^{\sqrt{3}/n} - 1}{\frac{\sqrt{3}}{n}} \cdot \frac{\frac{2}{n}}{\sin \frac{2}{n}} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$b) \lim_{n \rightarrow \infty} [\log(8+n) - \log(en)] = [\infty - \infty]$$

$$= \lim_{n \rightarrow \infty} \log\left(\frac{8+n}{en}\right) = \log\left(\lim_{n \rightarrow \infty} \frac{8+n}{en}\right) = \log \frac{1}{e} = -1$$

$$c) \lim_{n \rightarrow \infty} \frac{3n^2 + 2n^4 - 1302n(-1)^n}{n^2 + 1302n^5 \cos(n\pi)} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\left(\frac{3}{n^2} + 2 - \frac{1302(-1)^n}{n}\right)}}{n^3 \left(\frac{1}{n^3} + 1302(-1)^n\right)} = 0$$

$$3) a) \lim_{n \rightarrow \infty} n^6 \left(1 - \sqrt{\cos \frac{n}{n^3}}\right) = [\infty \cdot 0]$$

$$= \lim_{n \rightarrow \infty} n^6 \left(1 - \sqrt{\cos \frac{n}{n^3}}\right) \cdot \frac{1 + \sqrt{\cos \frac{n}{n^3}}}{1 + \sqrt{\cos \frac{n}{n^3}}} = \lim_{n \rightarrow \infty} \frac{1}{1 + \sqrt{\cos \frac{n}{n^3}}} \cdot \frac{1 - \cos \frac{n}{n^3}}{\frac{1}{n^6}} = \frac{1}{4}$$

$$b) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^{2015}}\right)^{2015n + 1302} = [1^\infty]$$

$$n^{2015} = m \quad n \rightarrow \infty \Rightarrow m \rightarrow \infty$$

$$= \lim_{m \rightarrow \infty} \underbrace{\left(1 + \frac{1}{m}\right)^m}_{\rightarrow e} \cdot \underbrace{\left(1 + \frac{1}{m}\right)^{1302}}_{\rightarrow 1} = e$$

$$4) a) \lim_{x \rightarrow \infty} \left(1 - \cos \frac{1}{x}\right) (1 + \sin(e^x)) \quad -1 \leq \sin(e^x) \leq 1$$

$$\Rightarrow 0 \leq 1 + \sin(e^x) \leq 2$$

$$\Rightarrow 0 \leq \left(1 - \cos \frac{1}{x}\right) (1 + \sin(e^x)) \leq \left(1 - \cos \frac{1}{x}\right) \cdot 2$$

↓  
0

th carabinieri

↓  
0

⇒ il limite fa zero

$$b) \lim_{x \rightarrow 1} \frac{\sin(x(x-1)^2)}{x^2 - x} = \left[\frac{0}{0}\right]$$

$$= \lim_{x \rightarrow 1} \frac{\sin(x(x-1)^2)}{x(x-1)} \cdot \frac{x-1}{x-1} = \lim_{x \rightarrow 1} \frac{\sin(x(x-1)^2)}{x(x-1)^2} \cdot \underbrace{(x-1)}_{\downarrow 0} = 0$$

$$c) \lim_{x \rightarrow \infty} \frac{\sin x}{\log x} \quad -1 \leq \sin x \leq 1$$

$$\Rightarrow \frac{-1}{\log x} \leq \sin x \cdot \frac{1}{\log x} \leq \frac{1}{\log x}$$

↓ th carabinieri ↓  
0

⇒ il limite fa zero

$$5) a) \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\arcsin(3x)} = \left[\frac{0}{0}\right]$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\arcsin(3x)} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2e^{2x}}{\sqrt{1-9x^2}} = \frac{2}{3}$$

$$b) \lim_{x \rightarrow 0} \frac{\log \cos x}{\sin 2x} = \left[\frac{0}{0}\right]$$

$$\lim_{x \rightarrow 0} \frac{-\sin x}{2 \cos x} = \lim_{x \rightarrow 0} \frac{1}{2} \cdot \frac{-\sin x}{\cos x \cdot \cos 2x} = 0$$

$$c) \lim_{x \rightarrow 0} \left(1 - \frac{x}{2}\right)^{\frac{1}{\sin 2x}} = [1^\infty]$$

$$\lim_{x \rightarrow 0} e^{\frac{1}{\sin 2x} \log\left(1 - \frac{x}{2}\right)} = e^{\lim_{x \rightarrow 0} \frac{\log\left(1 - \frac{x}{2}\right)}{\sin 2x}} \stackrel{H}{=} e^{\lim_{x \rightarrow 0} \frac{-\frac{1}{2}}{2 \cos 2x}} = e^{-1/4} = \frac{1}{\sqrt[4]{e}}$$

$$b) a) D \left[ \frac{(\sin x)^x}{\tan x} \right] = \frac{(D e^{x \log \sin x}) \tan x - (\sin x)^x (1 + \tan^2 x)}{\tan^2 x}$$

$$= \frac{(\sin x)^x \left[ \log \sin x + x \frac{\cos x}{\sin x} \right] \tan x - (\sin x)^x (1 + \tan^2 x)}{\tan^2 x}$$

$$b) D [x^7 \cos(7x) - x^5 \sin(5x) + 1] = 7x^6 \cos 7x - 7x^7 \sin 7x - 5x^4 \sin(5x) - 5x^5 \cos(5x)$$

$$8) a) \int \frac{1}{4x^2 + 1} dx = \int \frac{1}{4(z)^2} dz = \frac{1}{2} \int \frac{2}{1 + (2z)^2} dz = \frac{1}{2} \arctan(2z)$$

$$b) \int e^{2x} \sin x dx \quad \text{per parti} \quad \begin{array}{cc} e^{2x} & 2e^{2x} \\ \sin x & -\cos x \end{array}$$

$$= [-(\cos x) e^{2x}] + 2 \int e^{2x} \cos x dx = \begin{array}{cc} \text{per parti ancora} \\ e^{2x} & 2e^{2x} \\ \cos x & \sin x \end{array}$$

$$= -e^{2x} \cos x + 2e^{2x} \sin x - 4 \int e^{2x} \sin x dx$$

$$\Rightarrow \int e^{2x} \sin x dx = \frac{1}{5} e^{2x} (2 \sin x - \cos x)$$

$$7) \quad y = f(x) = x \log x$$

dominio:  $x > 0$  per il log.  $D = \{x > 0\} = (0, +\infty)$

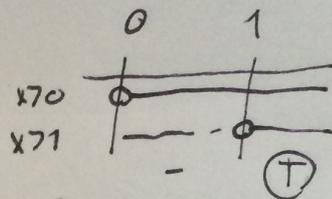
int. am.:  $x=0$  non si fa

$$y=0 \quad x \log x = 0 \quad x=0 \quad x$$

$$\log x = 0 \quad x=1 \quad \checkmark$$

segno:  $x \log x > 0 \quad x > 0$

$$\log x > 0 \quad x > 1$$



$\lim_{x \rightarrow 0^+} x \log x = 0$  confronto tra potenze e log.

derivata prima:  $y' = \log x + 1 \quad D = (0, +\infty)$

segno:  $y' > 0 \quad \log x + 1 > 0$

$$\log x > -1$$

$$x > e^{-1} = \frac{1}{e}$$

~~lim~~  $\lim_{x \rightarrow 0^+} y' = \lim_{x \rightarrow 0^+} (\log x + 1) = -\infty$

$$y' = 0 \quad \log x + 1 = 0 \quad \log x = -1 \quad x = e^{-1} = \frac{1}{e} \quad \text{è minimo assoluto}$$

$$f\left(\frac{1}{e}\right) = \frac{1}{e} \log \frac{1}{e} = \frac{1}{e} (-1) = -\frac{1}{e}$$

$y'' = \frac{1}{x}$   $D = \{x \neq 0\}$   $y'' > 0$  nel dominio di  $f$   
quindi  $f$  ha la concavità verso l'alto

$$\int_0^a x \log x \, dx = 0 \quad \text{per quale } a > 0?$$

$\int_0^a x \log x \, dx$  per parti

$$\begin{array}{l} \log x \quad \frac{1}{x} \\ x \quad \frac{x^2}{2} \end{array}$$

$$= \left[ \frac{x^2}{2} \log x \right]_0^a - \int_0^a \frac{x}{2} \, dx = \frac{a^2}{2} \log a - 0 - \left[ \frac{x^2}{4} \right]_0^a = \frac{a^2}{2} \log a - \frac{a^2}{4} =$$

non accettabile perché  $\alpha > 0$

$$= \frac{\alpha^2}{x} \left[ \log \alpha - \frac{1}{2} \right] = 0$$

$$\log \alpha = \frac{1}{2}$$

$$\alpha^2 = 0$$

$$\alpha = 0$$

non accettabile perché

$$\alpha > 0$$

$$\alpha = e^{1/2}, \quad \sqrt{e} \approx 1.65$$

$$f(x) = y$$

batto Nike

