Mini-workshop

Mathematical Models in Continuum Mechanics

Politecnico di Torino – January 20, 2020

Schedule and seminars abstracts

Venue

Aula Buzano
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Schedule

9:00–9:15 *Opening remarks*

9:15–10:00 David R. Owen: *Multiscale Geometry and the Enrichment of Continuum Mechanics*

A description of geometrical changes at submacroscopic levels can be used to enrich the mathematical infrastructure that underlies continuum mechanics. In this talk I will describe such a multiscale geometry based on structured deformations of continua, provide an elementary example of an enriched energetics that captures the features of yielding and hysteresis in single crystals, and describe an enriched field theory that is intended to model the mechanical behavior of lipid monolayers, biological membranes central to the physiology of the lungs.

10:00–10:45 Alfio Grillo: *Structural inhomogeneities and anomalous diffusion in tumour growth*

In this contribution, I will discuss some biomechanical aspects of tumour growth from two different perspectives. First, I will show how, through the determination of a suitably defined Riemannian curvature tensor, Differential Geometry offers a natural tool for describing the structural changes of a tumour [1, 2]. In particular, I will connect these concepts with the property of material homogeneity of a growing medium [3] and I will present some results in which growth is influenced by the curvature that it itself generates [4]. The second perspective addresses the issue of transport of chemical substances in a tumour. Within this context, I will show some preliminary results obtained under the hypothesis that the considered transport mechanisms involve the anomalous diffusion of the nutrient species feeding the tumour cells. These results are taken from an on-going study done in collaboration with Ariel Ramírez-Torres and Salvatore Di Stefano.

[1] A. Yavari and A. Goriely: *Weyl geometry and the nonlinear mechanics of distributed point defects*. Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **468** (2012), 3902-3922.

[2] A. Goriely: The mathematics and mechanics of biological growth. Springer, NewYork, 2016.

[3] M. Epstein: *Self-driven continuous dislocations and growth* in: M. G. Steinmann P. (Ed.), Mechanics of Material Forces. Advances in Mechanics and Mathematics, vol. 11, Springer, Boston, MA, 2005, pp. 129-139.

[4] S. Di Stefano, A. Ramírez-Torres, R. Penta, and A. Grillo: *Self-influenced growth through evolving material inhomogeneities*, Int. J. Non-Linear Mechanics **106** (2018), 174-187.

10:45–11:15 *coffee break*

11:15–12:00 Luca Lussardi: *Minimizers of the multiphase Canham-Helfrich functional*

In this seminar I will speak about the minimization of the Canham-Helfrich functional in presence of multiple phases. The problem is inspired by the modelization of heterogeneous biological membranes, which may feature variable bending rigidities and spontaneous curvatures. With respect to previous contributions, no symmetry of the minimizers is here assumed. Correspondingly, the problem is reformulated and solved in the weaker frame of oriented curvature varifolds. I will discuss lower semicontinuity, existence of single and multiphase minimizers under area and enclosed volume constrains, and regularity of minimizers. This is a joint work with K. Brazda and U. Stefanelli, both from University of Vienna.

12:00–12:45 José Matias: *Explicit integral representations of the relaxation of non-local energies for structured deformations*

The theory of structured deformation in the SBV setting developed by [1] only takes into account the linear dependance on jumps along the approximating sequences. In [2] a model was proposed toward capturing the non-linear dependence on the jumps. The idea was to modify the initial energy as follows: for each $r \in (0,1)$ let

$$F^r(u) := \int_0^1 W(\nabla u(x))\,\mathrm{d}x + \sum_{z\in S_u} \psi([u](z)) + \int_0^1 \Psi\Bigg(\sum_{z\in S_u\cap(x-r,x+r)} \frac{[u](z)}{2r}\Bigg)\mathrm{d}x,$$

and then undergo a relaxation process in the context of structured deformations followed by taking the limit as $r \to 0^+$. In this work we extend this model for any dimensions and different types of non-local energies Ψ. Moreover, we consider more general types of averaging of the jumps.
This is a joint work with M. Morandotti, D. R. Owen, and E. Zappale

[1] R. Choksi and I. Fonseca: *Bulk and interfacial energy densities for structured deformations of continua*. Arch. Rational Mech. Anal. **138** (1997), 37-103.

[2] G. Del Piero and D. R. Owen: *Structured Deformations: Part Two*. Quaderni dell'Istituto Nazionale di Alta Matematica, Gruppo Nazionale di Fisica Matematica, no. **58** (2000), 1-62.

12:45–14:30 *lunch break*

14:30–15:15 Marco Gherlone: *Mixed theories for the analysis of multilayered composite and sandwich structures*

Over the past three decades, composite materials and structures have been increasingly used in civil, automotive, and aerospace applications. Several structural theories have been proposed for the analysis of multilayered composite and sandwich structures. Since transverse shear deformation governs critical damage mechanisms (delamination), efforts to develop accurate predictions of transverse shear strains and stresses have been extensive.

Among the many available theories, the Refined Zigzag Theory (RZT) represents a good compromise between low computational cost and accuracy [1]. RZT enables accurate predictions of the in-plane displacements, strains, and stresses over a wide a range of multilayered structures. The transverse shear stresses obtained from constitutive equations are layer-wise constant. Although these transverse shear stresses are generally accurate in the average, layer-wise sense, they are nevertheless discontinuous at layer interfaces, and thus they violate the requisite interlaminar continuity of transverse stresses. Therefore, a mixed RZT ($\mathrm{RZT}^{(\mathrm{m})}$) has also been proposed that is based on the Reissner's mixed variational theorem [2-4]. One of the key aspects of the $\mathrm{RZT}^{(\mathrm{m})}$ formulation is the way transverse shear stresses are assumed.

Aim of the seminar is to provide an overview of RZT and $\mathrm{RZT}^{(\mathrm{m})}$ and a discussion on the several approaches (with related benefits and drawbacks) to the transverse shear stresses field assumption in mixed theories for the analysis of multilayered composite and sandwich structures.

[1] A. Tessler, M. Di Sciuva, and M. Gherlone: *A consistent refinement of first-order shear-deformation theory for laminated composite and sandwich plates using improved zigzag kinematics*. Journal of Mechanics of Materials and Structures **5**(2) (2010), 341-367.

[2] E. Reissner: *On a certain mixed variational theorem and a proposed application*. International Journal for Numerical Methods in Engineering **20**(7) (1984), 1366-1368.

[3] A. Tessler: *Refined Zigzag Theory for homogeneous, laminated composite, and sandwich beams derived from Reissner's mixed variational principle*. Meccanica **50**(10) (2015), 2621-2648.

[4] L. Iurlaro, M. Gherlone, M. Di Sciuva, and A. Tessler: *Refined Zigzag Theory for laminated composite and sandwich plates derived from Reissner's Mixed Variational Theorem*. Composite Structures **133** (2015), 809-817.

15:15–16:00 Lorenza D'Elia: *Homogenization of convolution-type functionals defined on general periodic domains*

We investigate the asymptotic behaviour of convolution-type functionals which are defined on general perforated domains. The corresponding Γ-limit is a functional represented in a local form and whose integrand is defined by a non-local cell problem formula. It is achieved by proving an extension theorem under the only assumption that the periodic set is connected. This is a joint work with Prof. A. Braides (Università di Roma Tor Vegata) and Prof. V. Chiadò Piat (Politecnico di Torino).

16:00–16:30 *coffee break*

16:30–17:15 Elvira Zappale: *Relaxation results for variational nonlocal problems*

Variational problems involving nonlocal supremal functionals

$$L^\infty(\Omega;\mathbb{R}^m) \ni u \mapsto \text{esssup}_{(x,y)\in\Omega\times\Omega} W(u(x),u(y)),$$

where $\Omega \subset \mathbb{R}^n$ is a bounded, open set and $W\colon \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a suitable function, are studied. Motivated by existence theory via the direct method, it is possible to identify a necessary and sufficient condition for L^∞-weak* lower semicontinuity of these functionals, namely, separate level convexity of a symmetrized and suitably diagonalized version of the supremands. More generally, it is shown that the supremal structure of the functionals is preserved during the process of relaxation. The proof relies substantially on the connection between supremal and indicator functionals. This allows to recast the relaxation problem into characterizing weak* closures of a class of nonlocal inclusions, which is of independent interest. To illustrate the theory, explicit relaxation formulas are given for examples of functionals with different multi-well supremands. On the basis of these results it is possible to provide explicit examples to show that the relaxation of functionals

$$L^p(\Omega) \ni u \mapsto \int_\Omega \int_\Omega W(u(x),u(y))\,dx\,dy,$$

where $\Omega \subset \mathbb{R}^n$ is an open and bounded set, $1 < p < \infty$ and $W\colon \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ a suitable integrand, is in general not of double-integral form. This proves an up to now open statement in [Pedregal, *Rev. Mat. Complut.* **29** (2016)] and [Bellido & Mora-Corral, *SIAM J. Math. Anal.* **50** (2018)].

Joint work with Carolin Kreisbeck.

17:15–17:30 *Closing remarks*