ESERCIZI

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1. Calcolare le seguenti derivate (semplificando il piú possibile le espressioni)

(1) $D[\sqrt{(x+a)(x+b)(x+c)}]$

(2) $D\left[\sin(x^2-5x+1)+\tan\frac{a}{x}\right]$.

(3) $D\left[\sin 3x+\cos\frac{x}{5}+\tan\sqrt{x}\right]$.

(4) $D\left[\sin(2x)\cos(3x)\right]$.

(5) $D\left[\sin\frac{x}{2}\sin(2x)\right]$.

(6) $D\left[2\tan^3\frac{x}{2}-6\tan\frac{x}{2}+3x\right]$.

(7) $D\left[\sin^2\frac{x}{3}\cot\frac{x}{2}\right]$.

(8) $D\left[\frac{1}{18}\sin^6 3x-\frac{1}{24}\sin^8 3x\right]$.

(9) $D\left[-\frac{1}{20}\cos(5x^2)-\frac{1}{4}\cos(x^2)\right]$.

(10) $D\left[\frac{\tan\frac{x}{2}+\cot\frac{x}{2}}{x}\right]$.

(11) $D\left[\sin(7x^2)+\sqrt[3]{1+x^2}\right]$.

(12) $D\left[x\log^3 x-3x\log^2 x+6x\log x-6x\right]$.

(13) $D\left[\sqrt{1+\log^2 x}\right]$.

(14) $D\left[\sqrt{\log x+1}+\log(\sqrt{x}+1)\right]$.

(15) $D\left[\log\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}+2\arctan\sqrt{\frac{1-x}{1+x}}\right]$.

(16) $D\left[2\log(2x-3\sqrt{1-4x^2})-6\arcsin(2x)\right]$.

(17) $D\left[\frac{xe^x\arctan x}{\log^5 x}\right]$.

(18) $D\left[(\tan 2x)^{\cot(x/2)}\right]$.

(19) $D\left[2\arcsin\frac{x-2}{\sqrt{6}}-\sqrt{2+4x-x^2}\right]$.

(20) $D\left[|x^2+1|\right]$.

(21) $D\left[|\sin x|+|\cos x|\right]$.

(22) $D\left[(\arctan x)^x\right]$.

(23) $D\left[2^{\arcsin(3x)}+(1-\arccos(3x))^2\right]$.

(24) $D\left[\frac{1}{\sqrt{b}}\arcsin\left(x\sqrt{\frac{b}{a}}\right)\right]$.

(25) $D\left[\dfrac{3x^2-1}{3x^3}+\log\sqrt{1+x^2}+\arctan x\right]$.

(26) $D\left[\log\dfrac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}+2\arctan\dfrac{x\sqrt{2}}{1-x^2}\right]$.

(27) $D\left[\exp(\arctan\sqrt{1+\log(2x+3)})\right]$.

(28) $D\left[\dfrac{1}{\sqrt{3}}\arctan\dfrac{x\sqrt{3}}{1-x^2}\right]$.

2. Calcolare i seguenti limiti con i teoremi di de l'Hôpital

(1) $\lim\limits_{x\to 0}\dfrac{\log\cos x}{\sin 2x}$ $[0]$.

(2) $\lim\limits_{x\to 0}\dfrac{x-\arctan x}{x^3}$ $[1/3]$.

(3) $\lim\limits_{x\to a}\dfrac{x^m-a^m}{x^n-a^n}$ $[m/n]$ se $m\neq n, a\neq 0$.

(4) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\arcsin(3x)}$ $[2/3]$.

(5) $\lim\limits_{x\to 5}\dfrac{\sqrt[3]{x}-\sqrt[3]{5}}{\sqrt{x}-\sqrt{5}}$ $[2/(3\sqrt[6]{5})]$.

(6) $\lim\limits_{x\to 0}\dfrac{\log\cos(ax)}{\log\cos(bx)}$ $[a^2/b^2]$.

(7) $\lim\limits_{x\to 0}\dfrac{e^{3x}-3x-1}{\sin^2(5x)}$ $[9/50]$.

(8) $\lim\limits_{x\to 0}\left(1-\dfrac{x}{2}\right)^{\frac{1}{\sin(2x)}}$.

(9) $\lim\limits_{x\to 2}\dfrac{\log(x^2-3)}{x^2+3x-10}$ $[4/7]$.

(10) $\lim\limits_{x\to 0}\dfrac{2-(e^x+e^{-x})\cos x}{x^4}$ $[1/3]$.

(11) $\lim\limits_{x\to 1}\dfrac{a^{\log x}-x}{\log x}$ $[\log a-1]$.

(12) $\lim\limits_{x\to 1}\dfrac{1-4\sin^2\frac{\pi}{6}x}{1-x^2}$ $[\pi\sqrt{3}/6]$.

(13) $\lim\limits_{x\to 0}\dfrac{\sin 3x-3xe^x+3x^2}{\arctan x-\sin x-x^3/6}$ $[18]$.

(14) $\lim\limits_{x\to 0}\dfrac{e^{a\sqrt{x}}-1}{\sqrt{\sin(bx)}}$ $[a\sqrt{b}/b]$.

(15) $\lim\limits_{x\to +\infty}\dfrac{e^2-\left(1+\frac{2}{x}\right)^x}{1/x}$ $[2e^2]$.

(16) $\lim\limits_{x\to 0}\dfrac{e^{\tan x}-e^x}{\tan x-x}$ $[1]$.

(17) $\lim\limits_{x\to 0}\dfrac{\sin(-2x)}{3x}$.

(18) $\lim\limits_{x\to 0^+}\dfrac{\log x}{\cot x}$ $[0]$.

(19) $\lim\limits_{x\to 0}\dfrac{\cot 5x}{\cot x}$ $[1/5]$.

(20) $\lim\limits_{x\to 0^+}\dfrac{\log\sin x}{\cot x}$ $[0]$.

(21) $\lim\limits_{x\to a^+}\dfrac{\log(x-a)}{\log(e^x-e^a)}$ $[1]$.

(22) $\lim\limits_{x\to +\infty}\dfrac{\log(1+e^{x^2})}{\cot\pi x}$ $[0]$.

(23) $\lim\limits_{x\to 1}\dfrac{\tan(\pi x/2)}{\log(1-x)}$ $[\infty]$.

(24) $\lim_{x\to 0^+} \dfrac{\log \sin 2x}{\log \sin x}$ $[1]$.

(25) $\lim_{x\to 0^+} \dfrac{\log x}{\log \sin x}$ $[1]$.

(26) $\lim_{x\to +\infty} \dfrac{\log \sqrt[3]{x^3+x^2}}{\log x}$ $[1]$.

(27) $\lim_{x\to 0} \dfrac{\log \sin ax}{\log \sin bx}$ $[1]$.

(28) $\lim_{x\to 0} \dfrac{\log x}{1+2\log \sin x}$ $[1/2]$.

(29) $\lim_{x\to 3} \dfrac{\cos x \log(x-3)}{\log(e^x-e^3)}$ $[\cos 3]$.

(30) $\lim_{x\to 0} \dfrac{\pi/x}{\cot(\pi x/2)}$ $[\pi^2/2]$.

3. Formule di Taylor e Mac Laurin

Scrivere i polinomi approssimanti dell'ordine n indicato e centrati in x_0 delle seguenti funzioni.

(1) $f(x) = x^3 - 2x^2 + 3x + 5, \quad n = 3, \quad x_0 = 2.$

(2) $f(x) = x^5 - 2x^4 + x^3 - x^2 + 2x - 1, \quad n = 5, \quad x_0 = 1.$

(3) $f(x) = x^4 - 5x^3 + x^2 - 3x + 4, \quad n = 4, \quad x_0 = 4.$

(4) $f(x) = x^3 + 3x^2 - 2x + 4, \quad n = 3, \quad x_0 = -1.$

(5) $f(x) = x^{10} - 3x^5 + 1, \quad n = 10, \quad x_0 = 1.$

(6) $f(x) = e^x, \quad n = 3, \quad x_0 = -1.$

(7) $f(x) = \log x, \quad n = 4, \quad x_0 = 1.$

(8) $f(x) = \dfrac{1}{x}, \quad n = 3, \quad x_0 = a.$

(9) $f(x) = xe^x, \quad n = 4, \quad x_0 = 0.$

(10) $f(x) = \dfrac{x}{x-1}, \quad n = 4, \quad x_0 = 2.$